	1.0	4.5	2.8	2.5
		5.0		
		5.6	3.2	2.2
		6.3	3.6	
	1.1		4.0	2.0
				1.8
	1.25		1.4	1.6

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

② LEVEL II

NPS55-79-020

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Technical rept.,

STATISTICAL METHODS OF PROBABLE USE
FOR UNDERSTANDING REMOTE SENSING DATA.

by

Donald P. Gaver

40

October 1979

Prepared for:

Naval Postgraduate School
Monterey, Ca. 93940

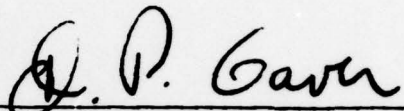251 450

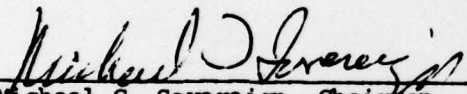79 11 16 011

Naval Postgraduate School
Monterey, California

Rear Admiral T. F. Dedman   Jack R. Borsting
Superintendent   Provost
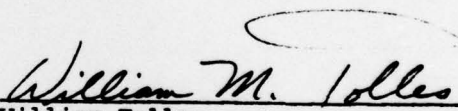

This report was prepared by:

D. P. Gaver, Professor
Department of Operations Research

Reviewed by: Released by:

Michael G. Sovereign, Chairman   William Tolles
Department of Operations Research   Dean of Research

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55-79-020	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Statistical Methods of Probable Use for Under- standing Remote Sensing Data		5. TYPE OF REPORT & PERIOD COVERED Technical
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) D. P. Gaver		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		12. REPORT DATE October 1979
		13. NUMBER OF PAGES 38
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Statistics	Censoring
Robustness	Smoothing
Remote Sensing	Extreme Values
Regression	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report outlines several new statistical approaches to data problems likely to be encountered when remote sensing methods are used. The methods described are robust regression analyses, smoothing, and modeling and estimation of ice pressure ridge characteristics.

STATISTICAL METHODS OF PROBABLE USE
FOR UNDERSTANDING REMOTE SENSING DATA

Donald P. Gaver

Naval Postgraduate School
Monterey, CA 93940

1. INTRODUCTION

Statistical methodology has long been a familiar tool for use in understanding our natural environment. Classical examples of applications of statistics are seen in weather forecasting, in evaluation of attempts at weather modification by cloud seeding, and in descriptions of the fluctuations in the sea surface. Now the accessibility of new and extensive data from a variety of remote sensing sources, such as earth orbiting and geostationary satellites, again calls for the development and application of appropriate statistical methodology. Classical methods of statistics and of probability modeling frequently must be adapted to the new needs. The process of adaptation will proceed most efficiently if statisticians work cooperatively with the scientists actually obtaining data and studying the associated natural phenomena. Conferences such as PRIMARS I are of great value in promoting the necessary interchange of information and the stimulus to approach novel and difficult problems in a realistic manner.

This paper describes new approaches to the analysis of data, in particular to quite "noisy" data of the sort that is likely to be encountered when observing the natural environment. The descriptions given will necessarily be brief, but an attempt will be made to show how the methods and viewpoints presented may be applied to problems arising in remote sensing.

1

2. ROBUST METHODOLOGY: REQUIREMENTS AND POSSIBILITIES

Many scientists who have closely examined real data have encountered occasional, or even frequent, anomalous behavior. Apparent anomalies in data may be with respect to either

(a) preconceptions as to "proper" data behavior, these perhaps being buttressed by (physical) theory, or

(b) the nature of the general pattern of the data, especially those data points in the immediate neighborhood, e.g., in time or space.

2.1. Plots

In simple circumstances graphical plots will quickly reveal those points that are blatant anomalies. For instance suppose that one wishes to investigate data concerning the relationship between wind velocity and whitecap cover in the ocean. Theory may suggest a specific relationship, e.g. that white-cap cover, C, be nearly a cubic function of wind velocity v, so that it will be tempting to plot C $\underline{vs}$ v and note an appearance as shown on Fig. 1; there solid black dots represent (simulated) raw data. Since the eye finds it difficult to distinguish curves of the form $C = \alpha v^2$, $C = \alpha v^3$, $C = \alpha v^{7/2}$, etc., from one another, and yet is sensitive to departures from linearity, a graph of C $\underline{vs}$ v^3 suggests itself, but is not included here. A plot on log-log paper may be still better.

As presented, the data conforms in general to the theorized relation-
ship or scaling, with the obvious exception of the circled point to
the right. Such an anomalous point, or points, represents a challenge
both to statistical technology and to the ultimate user of the data.

Statistical technology assumes the responsibility for revealing
the presence of such points, and, if possible, for providing a
meaningful and useful summary of the remaining points. It falls
to the consumer or ultimate user of the data, preferably with
the help of a subject-matter specialist (physicist or oceanographer)
to interpret the apparently anomalous maverick--or exotic, or
outlying--data point: is it

  (i) an evidence of the failure of the relation $C = \alpha v^3$, say
       for large velocities,

or is it

(ii) an outright error in data recording, and to be disregarded?

being just two possible options.

        Note that simple graphs are invaluable for pointing out
extreme outliers in simple, one explanatory variable, situations.
If more variables are required, informative plots are more diffi-
cult without the use of more statistical technology. We next
show that classical, least-squares, technology may be quite mis-
leading, but that replacements are available. See Mosteller and
Tukey (1977), abbreviated MT hereafter.

3

2.2. Fits and Residual Plots

Suppose that one wishes to summarize data such as that in Fig. 1 by fitting the relationship $C = \alpha v^3$, i.e., determining the parameter α from the data. The classical and automatic way of doing so is to apply least squares; computer programs are universally available, even for handheld calculators (the TI 59, or HP 67). What are we likely to find? A least-squares line (treating $w = v^3$ as the independent variable presents $C = \alpha w$; one can also plot and fit $C^{1/3} = (a)^{1/3} v$, and there may be reasons for this choice) is quite apt to fatally misrepresent the situation, responding much too sensitively to the single (here encircled) outlying value, and straying systematically away from the main body of the data; see the points represented by o in Fig. 1.

An alternative method for fitting, described in MT, is less susceptible to outlier influence--is far more robust to departures from basic assumptions--than is the ordinary least squares (OLS) method. This new method, termed biweight fitting, is carried out by a procedure that uses the OLS computation iteratively. In the course of the computations weights are automatically developed that reduce the influence of the encircled value of Fig. 1, permitting the fit to more closely approximate the main body of the data. We now describe and illustrate the biweight fitting procedure as it is adapted to the problem of determining the parameter α in the relation y_i vs αx_i.

4

Biweight Fitting Calculation

(1) Compute the kth ($k = 1,2,3,\ldots$) iterative estimate of α, denoted by $\alpha^{(k)}$ by solving

$$\sum_{i=1}^{n} (y_i - \alpha^{(k)} x_i) x_i w_i^{(k-1)} = 0 \; ,$$

to obtain

$$\alpha^{(k)} = \frac{\displaystyle\sum_{i=1}^{n} y_i x_i w_i^{(k-1)}}{\displaystyle\sum_{i=1}^{n} x_i^2 w_i^{(k-1)}} \; ,$$

(2) the weights, $w_i^{(k-1)}$, are of this form:

$$w_i^{(k-1)} = \left[1 - \left(\frac{y_i - \alpha^{(k-1)} x_i}{cS^{(k-1)}} \right)^2 \right] \quad \text{if } (\cdot) \leq 1$$

$$= 0 \qquad\qquad\qquad\qquad \text{if } (\cdot) > 1$$

where $(\cdot)$ refers to the term $[(y_i - \alpha^{(k-1)} x_i)/S^{(k-1)}]$; $S^{(k-1)}$ is a scale factor (robust replacement for the standard deviation) that may be computed in the following manner.

(3) The $k-1^{st}$ iterated value of the scale factor is

$$S^{(k-1)} = \text{median}\{|y_i - \alpha^{(k-1)} x_i|\},$$

c being a constant of value 6, or 9;

5

(4) the first value, $\alpha^{(1)}$, of the iterative sequence can be obtained by equalizing all weights $(w_i^{(0)} = 1)$, which is equivalent to OLS; alternatively, one can utilize a "robust start," suggestions for which can be found in MT.

The iteration is carried on until the difference between successive values is small; usually 4 to 8 iterations is sufficient. The resulting α-estimate can be denoted by $\hat{\alpha}$.

Following the fitting it is informative to plot the residual values:

$$r_i = y_i - \hat{\alpha}x_i \equiv y_i - \hat{y}_i , \qquad i = 1,2,\dots,n,$$

$\hat{y}_i$ being shorthand for the predicted y value. In case there is a single outlier, as in Fig. 1, the fitted line will tend to hug the major point cloud, and a histogram of the residuals will dramatically reveal the presence of the outlier, suggesting further investigation. A plot of r_i vs y_i is also useful. See MT for further suggestions.

2.3. Numerical Illustration

The following are a set of (simulated) whitecap percentages and corresponding wind velocities. Alongside are values for white cap coverage estimated by OLS and by the biweight procedure.

6

Velocity	Cover ("Actual")	Cover (OLS Estimate)	Cover (Robust Estimate)
2	0.011	0.011	0.0067
7	0.63	0.49	0.29
10	1.30	1.43	0.84
15	3.89	4.82	2.83
18	2.89	8.33	4.88
21	8.16	13.2	7.76
24	25.7	19.7	11.6

It is clear from the above table, and perhaps clearer from Fig. 1, that the OLS solution, in its attempt to fit the point $C(24) = 25$, systematically and considerably over-estimates the points at $v = 15$ and above. The biweight estimator performs much better, allowing a closer fit to all data other than $C(24)$. A residual plot brings attention to bear on that point.

Since the values of "Actual Cover" were actually constructed by forming $0.008v^3$ and adding Gaussian random noise with value proportional to $C(v)$, and since the sequence of values of $0.008v^3$ were 0.0064, 0.27, 0.80, 2.7, 4.67, 7.41, 11.06, we cannot fault the manner in which the biweight procedure functioned in this example and are encouraged to use it more widely.

2.4. Possible Application to Remote Sensing Data

In a paper in this conference proceedings by Depriest (1979), and in Fleming (1979), a problem arising from partial cloud cover contamination of remote sensing data is described and addressed. This problem has the following origin. A series of measurements are made on a physical quantity (sea surface temperatures) but are contaminated. That is, in the case of sea surface temperatures, if no clouds are present the measurements are approximately normally distributed around μ (the true temperature). However, if clouds are present a fraction of the measurements are made artificially smaller, cloud temperatures being lower than those at earth surface. The problem is to estimate μ. Techniques for doing so are described by Depriest (1979) and by Fleming (1979). We describe a possible alternative approach that uses robust regression. Operational characteristics of the two procedures have not yet been compared.

(1) Arrange the measurements in order: $y_1 < y_2 < y_3 < \cdots < y_{n-1} < y_n$. The largest observations may well appear similar to the largest order statistics of a normal distribution with (unknown) mean μ and standard deviation σ (sometimes assumed known, although caution is in order), while the smaller ones are likely to depart systematically.

(2) Carry out a preliminary plot of

$$y_k \ \underline{vs} \ \phi^{-1}(\frac{k}{n+1}) \ , \qquad k = n, \ n-1, \ n-2, \ \ldots$$

8

where $\phi^{-1}(p)$ is the inverse function of the unit normal: recall that if

$$\phi(y) = \int_{-\infty}^{y} \exp(-\tfrac{1}{2} z^2) \; \frac{dz}{\sqrt{2\pi}} \, ,$$

is the unit normal distribution, then the solution of the equation $\phi(y) = p$ gives

$$y(p) = \phi^{-1}(p) ;$$

ϕ, and hence ϕ^{-1}, are widely tabulated. Alternatively, use Arithmetic Probability Paper. If y_k is an ordered observation from a normal population, then the plot should appear straight, while a systematic departure from linearity indicates a departure from normality. Suppose departures begin to occur at $k = D$; sometimes D may be greater than $n/2$. One may first eye-fit a straight line to the points $k = n, n-1, \ldots, D$. Then $\hat{y}_{n/2} = \hat{\mu}$ (estimated temperature). i.e. the value of the fitted line at $n/2$ should give a reasonable value for μ.

(3) Going further in a formal direction, one may wish to fit a line to the data points. Here a biweight fit should behave well, tending to be oblivious to spurious (cloud contamination) points. One can proceed to fit the relation

$$y_k \; \underline{vs} \; \mu + \sigma x_k$$

with

$$x_k = \phi^{-1}(\frac{k}{n+1}) \, , \qquad k = n, n-1, n-2, \ldots ;$$

9

a start using the eye-fit to points $k = n, n-1, \ldots , D$ may be worthwhile. Finally, quote the estimate

$$\hat{\mu} = \text{med} \ \hat{y}_k = \frac{1}{2} (\hat{y}_{n/2} + \hat{y}_{(n/2)+1}), \qquad n \text{ even};$$

$$= \hat{y}_{(n+1)/2}, \qquad n \text{ odd} .$$

where

$$\hat{y}_k = \hat{\mu} + \hat{\sigma} x_k .$$

The above procedure seems worth further investigation and refinement. One important step may be to adjust for the effect of correlation between order statistics when carrying out the regression.

3. SMOOTHING DATA

If one plots certain environmental data, e.g. monthly total rainfall, or perhaps daily maximum temperature, at a particular location, systematic regularities seem to appear, but may be masked by noise. Often there is a seasonal pattern, i.e. one that is roughly cyclic in nature. Attempts to fit such a pattern with polynomials is doomed to failure, and selection of a set of sines and cosines that does well (Fourier series) may lead to many terms. Some method of smoothing the original series that lays bare the regularities is to be desired. After such is made available, one can study the residuals around it. Spectral analysis or some such formal procedure may then be of use.

Classical smoothing procedures involve some form of moving average, and are susceptible to the python-swallowing-the pig difficulty: imagine using the linear smoothing operation

$$sy_t = \frac{y_{t-1} + y_t + y_{t+1}}{3}$$

on the y_t series

t	1	2	3	4	5	6	7	8	9	10
y_t	11	9	7	8	8	29	10	8	6	9
sy_t	⑪	9	8	7.7	15	15.7	15.7	8	7.7	⑩
Ry_t	⑪	9	8	8	8	10	10	8	8	⑩

11

clearly the entry of 29 at $t = 5$ into the smoothed value
series Sy_t gives a serious distortion, travelling as it does
in partially digested form through the next two terms of the
smoothed series. If further smoothing is attempted the bulge
is reduced slightly, but spreads out in time.

On the other hand, the non-linear operation of taking
running medians, as suggested by Tukey (1977), performs effec-
tively (in both cases circled and values are copied from the
original series; more sophisticated procedures can be invented
as well). The last row in the table, labelled Ry_t, gives the
result of this robust smoothing; note that it behaves in an
intuitively appealing manner, essentially ignoring the outlying
value 29. Further steps can be taken to improve the "smooths,"
but we refer to Tukey (1977) Chapters 7 and 16 for details.

Two further points may be made. The first is that the
analysis of a sequence of data points, and their projection or
forecasting in space or time, should not end with providing a
smoothed or averaged version. Examination of the remaining vari-
ation, e.g. the sequence $y_t - Ry_t$, called the "rough" by Tukey
may well be rewarding; presence and suggestiveness of various
outliers is much more evident in the rough (residuals) sequence
than in the original sequence of data points. Secondly, the
procedure described for smoothing simple sequences of data points
must be adapted to planar (two-dimensional) data; some work has
been done, but much remains.

4. STOCHASTIC MODELING OF ICE PRESSURE RIDGES

The dynamics of ice formation in the Earth's cold regions results in the development of irregular ice pile-ups, or pressure ridges. These ridges occur in an apparently random fashion in space; in fact the following regularities are observed by remote sensing methods (courtesy of Dr. W. Weeks, in a seminar at the Naval Postgraduate School, Monterey, California, Winter, 1979; see also Weeks, et al. (1979)):

1) Along a sampling line (e.g. airplane flight path, or straight submarine track) ice ridges seem to appear in accordance with a stationary Poisson process, so if $R(x)$ is the number of such ridges encountered over a distance x, then approximately

$$P\{R(x) = n\} = e^{-\lambda x} \frac{(\lambda x)^n}{n!} , \qquad n = 0,1,2,\ldots$$

where $\lambda > 0$ is the density of ice ridges.

2) The probability distribution of ridge "sail heights" (or "keel depths") may be approximated by the forms $F(y) = 1 - e^{-\mu y}$, or $1 - e^{-\nu y^2}$; the best-fitting distribution may well depend upon the method of observation (averaging properties).

For further details see work referenced in Weeks et al. (1979).

Now it may be of interest to compute the distribution of the maximum sail height, or keel depth, that one is to encounter over a course of length x. This is very simple, given the particular distributions of sail number and size and furthermore

13

assuming independent between ridge heights. Let $\bar{H}(x)$ be the maximum sail height; then

$$P\{\bar{H}(x) \leq y\} = \sum_{n=0}^{\infty} e^{-\lambda x} \frac{(\lambda x)^n}{n!} [F(y)]^n$$

since all of the Poisson-distributed heights must be less than y in order for the maximum to be below y.

Sum out to obtain

$$P\{\bar{H}(x) \leq y\} = \exp\{-\lambda x[1-F(y)]\}$$

Depending upon which distribution is picked for ridge heights, we get

a) $P\{\bar{H}(x) \leq y\} = \exp(-\lambda x e^{-\mu y})$

b) $P\{\bar{H}(x) \leq y\} = \exp(-\lambda x e^{-\nu y^2})$

These closely resemble classical extreme value distributions. Note that if logs are taken simplicity occurs:

a') $\ln P\{\bar{H}(x) \leq y\} = -\lambda x e^{-\mu y}$;

$\ln(-\ln P\{\bar{H}(x) \leq y\}) = \ln(\lambda x) - \mu y$

b') $\ln P\{\bar{H}(x) \leq y\} = -\lambda x e^{-\nu y^2}$;

$\ln(-\ln P\{\bar{H}(x) \leq y\}) = \ln(\lambda x) - \nu y^2$.

14

If either of these formulas are to be used for practical purposes, values of the parameters must be obtained. In order to estimate parameters λ, μ, ν in the above models from data one naturally thinks of the method of maximum likelihood. Suppose that we have observed $R(x) = n$ ridges of heights y_1, y_2, $\cdots$, y_n. Then the maximum likelihood estimates are

$$\hat{\lambda} = \frac{x}{n} \ , \quad \hat{\mu} = \frac{1}{\bar{y}} \ , \quad \hat{\nu} = \frac{1}{\overline{y^2}}$$

where as usual we have put

$$\overline{y^k} = \frac{1}{n} \sum_{i=1}^{n} y_i^k$$

Hence our estimates are of the form

a") $\quad$ est $\ell n(-\ell n \ P\{\bar{H}(x) \le y\}) = \ell n \ \hat{\lambda} + \ell n \ x - \hat{\mu}y$

b") $\quad$ est $\ell n(-\ell n \ P\{\bar{H}(x) \le y\}) = \ell n \ \hat{\lambda} + \ell n \ x - \hat{\nu}y^2$

If rather large samples are available and if distributional assumptions are well satisfied one may feel comfortable with conventional standard errors based on Fisher information and normality; see Cramér (1946). On the other hand, it is of interest to apply the <u>jackknife</u> <u>technique</u> (see R. G. Miller (1974) for a review) to obtain estimates of the variance of estimate due particularly to the ridge heights. To carry out the calculation, (i) compute $\hat{\nu}_{all} = n/\overline{y^2}$; then (ii) compute

$$\hat{\nu}_{(-j)} = \frac{n-1}{y_1^2 + y_2^2 + \cdots + y_{j-1}^2 + 0 + y_{j+1}^2 + \cdots + y_n^2}$$

15

for $j = 1, 2, \ldots, n$; then (iii) compute the pseudovalues $\hat{\nu}_j = n\hat{\nu}_{all} - (n-1) \hat{\nu}_{(-j)}$, and (iv) average to obtain a jackknifed point estimate $\hat{\nu}_{JK} = (1/n) \sum_{j=1}^{n} \hat{\nu}_j$, and its variance

$$\frac{1}{r} \left[\frac{1}{n-1} \sum_{j=1}^{n} (\hat{\nu}_j - \hat{\nu}_{JK})^2 \right] = s^2_{\hat{\nu}_{JK}} .$$

Then we can estimate the standard error of the probability prediction, e.g. b") by computing

$$S.E. = (Var[est \; \ell n(-\ell n \; P\{\bar{H}(x) \leq y\})])^{1/2}$$

$$\simeq \left(\frac{1}{\hat{\lambda}_{JK}} s^2_{\hat{\nu}_{JK}} + y^2 \; s^2_{\hat{\nu}_{JK}} \right)^{1/2}$$

A similar calculation is easily performed for model a); details are omitted. From the above results, approximate confidence intervals may be constructed for the probability of encountering a (maximum) ridge sail height less than y in magnitude.

Fairly recent theoretical results of Efron and Hinkley (1978) suggest that if a traditional maximum likelihood approach is taken, one is better off using observed Fisher information rather than expected Fisher information in order to establish an approximate standard error in either case a) or b). However, work of Reeds (1978) suggests that use of the jackknife in conjunction with maximum likelihood yields results that tend

16

to be rather independent of the basic model chosen. Both of these suggestions must be validated by further work, a good deal of which will necessarily involve Monte Carlo simulation. Such work should be of great importance and interest to those who must assess the probabilities of extreme, rare, events, and who furthermore wish to provide some reasonably valid estimates of the error of their estimates.

17

REFERENCES

Cramér, H. (1946), _Mathematical Methods of Statistics_, Princeton Univ. Press, Princeton, N.J.

Depriest, D. (1979), "Consideration   using a truncated normal distribution for remote sensing data." Paper presented at PRIMARS-1 Conference.

Efron, B., and Hinkley, D. (1978). "Assessing the accuracy of the maximum likelihood estimator: observed versus expected Fisher information," _Biometrika_ 65, No. 3, pp. 457-488.

Fleming, H. E. (1979). Application of the truncated normal distribution technique to the derivation of sea surface temperatures," to appear in Remote Sensing of Atmospheres and Oceans (1980), ed. by A. Deepak; Academic Press.

Miller, R. G. (1974). "The Jackknife--a review." _Biometrika_ 61, No. 1, pp 1-16.

Mosteller, F., and Tukey, J. W. (1977), _Data Analysis and Regression_. Addison-Wesley Publishing Co., Reading, Mass.

Reeds, J. A. (1978). "Jackknifing maximum likelihood estimates." _Annals of Statistics_ 6, No. 4, pp. 727-739.

Tukey, J. W. (1977). _Exploratory Data Analysis_, Addison-Wesley Publishing Co., Reading, Mass.

Weeks, N. F., Tucker, W. B., Frank, M., and Fungcharoen, S. (1979). "Characterization of the surface roughness and floe geometry of the sea ice over the continental shelves of the Beaufort and Chukchi seas." In "Sea Ice Processes and Models, Proc. AIDJEX/ICSI Sympos." (R.S. Pritchard, ed.), University of Washington Press (in press).
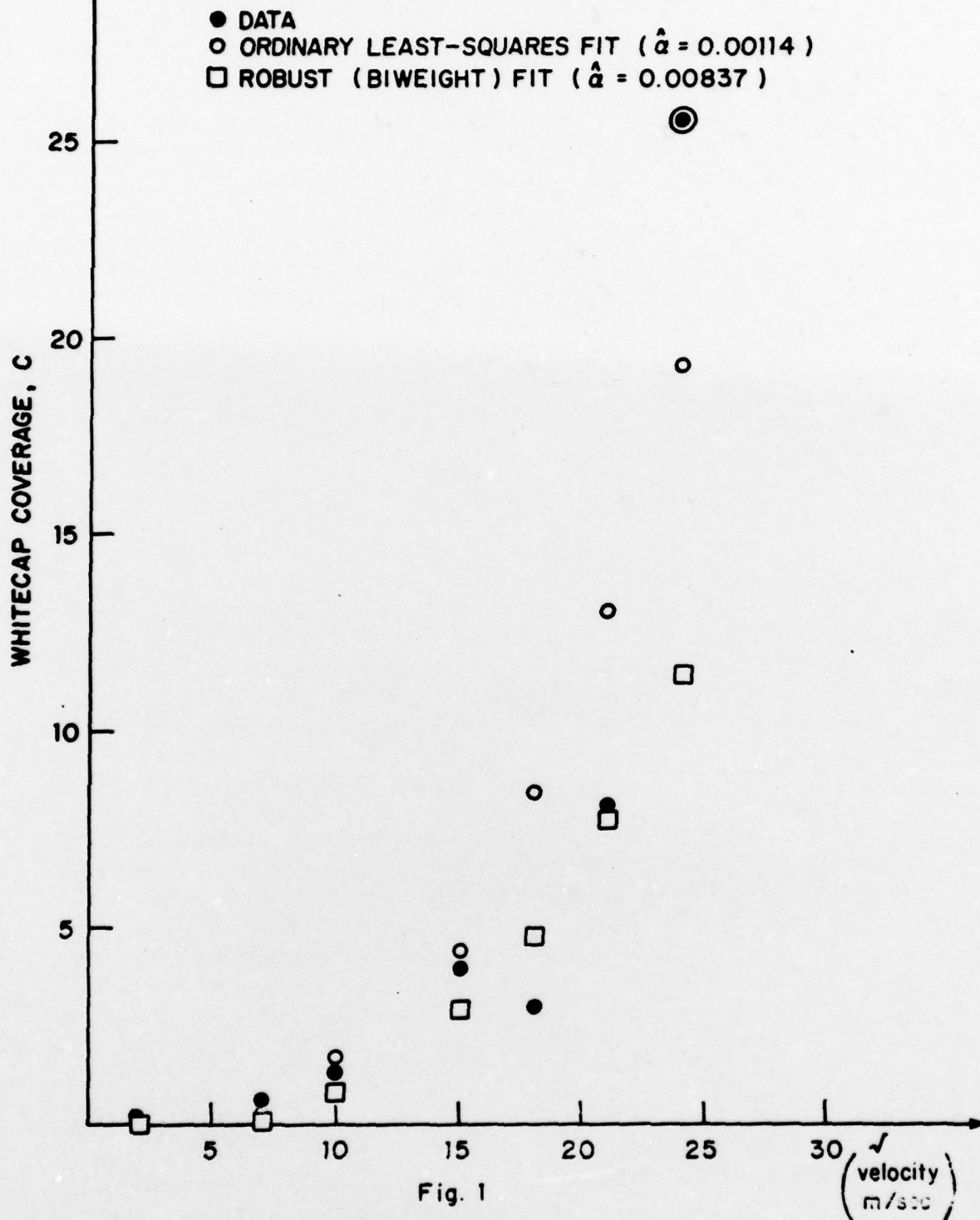
WHITECAP COVERAGE vs. WIND SPEED
(SIMULATED DATA)

● DATA
○ ORDINARY LEAST−SQUARES FIT ($\hat{\alpha}$ = 0.00114)
□ ROBUST (BIWEIGHT) FIT ($\hat{\alpha}$ = 0.00837)

Fig. 1

INITIAL DISTRIBUTION LIST

No. of Copies

	No. of Copies
STATISTICS AND PROBABILITY PROGRAM OFFICE OF NAVAL RESEARCH CODE 436 ARLINGTON VA 22217	1
OFFICE OF NAVAL RESEARCH NEW YORK AREA OFFICE 715 BROADWAY - 5TH FLOOR ATTN: DR. ROBER GRAFTON NEW YORK, NY 10003	1
DIRECTOR OFFICE OF NAVAL RESEARCH BRANCH OFF 536 SOUTH CLARK STREET ATTN: DEPUTY AND CHIEF SCIENTIST CHICAGO, IL 60605	1
LIBRARY NAVAL OCEAN SYSTEMS CENTER SAN DIEGO CA 92152	1
NAVY LIBRARY NATIONAL SPACE TECHNOLOGY LAB ATTN: NAVY LIBRARIAN BAY ST. LOUIS MS 39522	1
NAVAL ELECTRONIC SYSTEMS COMMAND NAVELEX 32C NATIONAL CENTER NO. 1 ARLINGTON VA 20360	1
DIRECTOR NAVAL RESEARCH LABORATORY ATTN: LIBRARY (ONRL) CODE 2029 WASHINGTON, D.C. 20375	1
TECHNICAL INFORMATION DIVISION NAVAL RESEARCH LABORATORY WASHINGTON, D. C. 20375	1

OFFICE OF NAVAL RESEARCH   1
SAN FRANCISCO AREA OFFICE
760 MARKET STREET
SAN FRANCISCO CALIFORNIA 94102

TECHNICAL LIBRARY   1
NAVAL ORDNANCE STATION

INDIAN HEAD MARYLAND 20640

NAVAL SHIP ENGINEERING CENTER   1
PHILADELPHIA
DIVISION TECHNICAL LIBRARY
PHILADELPHIA PENNSYLVANIA 19112

BUREAU OF NAVAL PRESONNEL   1
DEPARTMENT OF THE NAVY
TECHNICAL LIBRARY
WASHINGTON D. C. 20370

PROF. M. ABDEL-HAMEED   1
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF NORTH CAROLINA
CHARLOTTE
NC
                              28223

PROF. T. W. ANDERSON   1
DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY

STANFORD , CALIFORNIA 94305

PROF. F. J. ANSCOMBE   1
DEPARTMENT OF STATISTICS
YALE UNIVERSITY
NEW HAVEN
CONNECTICUT 06520

PROF. L. A. ARDIAN   1
INSTITUTE OF INDUSTRIAL
ADMINISTRATION
UNION COLLEGE
SCHENECTADY ,
NEW YORK 12308

PRCF. C. R. BAKER   1
DEPARTMENT CF STATISTICS
UNIVERSITY OF NOTRH CAROLINA
CHAPEL HILL,
NORTH CAROLINA
27514

PRCF. R. E. BECHHOFER   1
CEFARTMENT CF CPERATICNS RESEARCH
CCRNELL UNIVERSITY
ITHACA
NEW YORK 14850

PRCF. N. J. BERSHAC   1
SCHOOL CF ENGINEERING
UNIVERSITY CF CALIFCRNIA
IRVINE ,
CALIFCRNIA
92664

P. J. BICKEL   1
CEFARTMENT OF STATISTICS
LNIVERSITY CF CALIFCRNIA

BERKELEY , CALIFCRNIA

S4720

FROF. F. W. BLOCK   1
DEPARTMENT CF MATHEMATICS
UNIVERSITY OF PITTSBURGH
FITTSBURGH
FA

15260

PROF. JOSEPH BLUM   1
DEPT. OF MATHEMATICS, STATISTICS
   AND COMPLTER SCIENCE
THE AMERICAN UNIVERSITY
WASHINGTON
CC   20016

PROF. R. A. BRADLEY   1
DEFARTMENT CF STATISTICS
FLCRIDA STATE LNIVERSITY

TALLAHASSEE , FLCRIDA   32306

FROF. R. E. BARLOW   1
OPERATICNS RESEARCH CENTER
CCLLEGE CF ENGINEERING
UNIVERSITY CF CALIFCRNIA
BERKLEY
CALIFCRNIA 94720

MR. C. A BENNETT   1
NAVAL COASTAL SYSTEMS LABCRATORY
CCDE P761
FANAMA CITY,
FLCRIDA
32401

22

PRCF. L. N. BHAT
COMPUTER SCIENCE / OPERATICNS
RESEARCH CENTER
SOUTHERN METHODIST UNIVERSITY
DALLAS ,
TEXAS 75275   1

PRCF. W. R. ELISCHKE
DEPT. OF QUANTITATIVE
BUSINESS ANALYSIS
UNIVERSITY CF SOUTHERN CALIFORNIA
LOS ANGELES, CALIFORNIA
                              90007   1

CR. DERRILL J. BCRDELON
NAVAL UNCERWATER SYSTEMS CENTER
COCE 21
NEWPORT
RI
                              02840   1

J. E. BOYER JR
CEPT. OF STATISTICS
SOUTHERN METHODIST UNIVERSITY
DALLAS
TX
                              75275   1

DR. J. CHANDRA
U. S. ARMY RESEARCH
P. O. BOX 12211
RESEARCH TRIANGLE PARK ,
NORTH CARCLINA
27706   1

PRCF. H. CHERNCFF
DEPT. CF MATHEMATICS
MASS INSTITUTE CF TECHNOLOGY
CAMBRIDGE,
MASSACHUSETTS 0213S   1

FFOF. C. CERMAN
DEFARTMENT CF CIVIL ENGINEERING
AND ENGINEERING MECHANICS
COLUMEIA UNIVERSITY
NEW YCRK
NEW YORK   10027   1

PRCF. R. L. DISNEY
VIRGINIA PCLYTECHNIC INSTITUTE
 AND STATE LNIVERSITY
DEFT. CF INCUSTRIAL ENGINEERING
 AND OPERATICNS RESEARCH
BLACKSBURG, VA   24061   1

DISTRIBUTION LIST

MR. GENE H. GLEISSNER   1
APPLIED MATHEMATICS LABORATORY
DAVID TAYLOR NAVAL SHIP RESEARCH
AND DEVELOPMENT CENTER
BETHESDA
MD   20084

PROF. S. S. GUPTA   1
DEPARTMENT OF STATISTICS
PURDUE UNIVERSITY
LAFAYETTE
INDIANA 47907

PROF. D. L. HANSON   1
DEPT OF MATH. SCIENCES
STATE UNIVERSITY OF NEW YORK,
    BINGHAMTON
BINGHAMTON
NY   13901

PROF. M. J. HINICH   1
DEPARTMENT OF ECONOMICS
VIRGINIA POLYTECHNIC INSTITUTE
AND STATE UNIVERSITY
BLACKSBURG ,
VIRGINIA 24061

Dr. A. Deepak   1
Institute of Optics and Remote Sensing
Hampshire, VA

Dr. D. Depriest,   1
ONR, Code 102B
800 N. Quincy Street
Arlington, VA 22217

Mr. F. Diemer   1
ONR, Code 102B
800 N. Quincy Street
Arlingon, VA 22217

Professor P. B. Fellgett   1
Dept. of Engineering and Cybernetics
University of Reading
3 Earley Gate
Whiteknights, Reading RG6 2AL
ENGLAND

PROF. W. M. HIRSCH   1
INSTITUTE OF MATHEMATICAL SCIENCES
NEW YORK UNIVERSITY
NEW YORK
NEW YORK 10453

PROF. D. L. IGLEHART   1
DEPARTMENT OF OPERATIONS RESEARCH
STANFORD UNIVERSITY
STANFORD,
CALIFORNIA
94350

PROF. J. B. KADANE   1
DEPARTMENT OF STATISTICS
CARNEGIE-MELLON
PITTSBURGH,
PENNSYLVANIA
15213

DR. RICHARD LAU   1
DIRECTOR
OFFICE OF NAVAL RESEARCH BRANCH OFF
1030 EAST GREEN STREET
PASADENA
CA   91101

DR. A. R. LAUFER   1
DIRECTOR
OFFICE OF NAVAL RESEARCH BRANCH OFF
1030 EAST GREEN STREET
PASADENA
CA   91101

PROF. M. LEADBETTER   1
DEPARTMENT OF STATISTICS
UNIVERSITY OF NORTH CAROLINA
CHAPEL HILL
NORTH CAROLINA 27514

DR. J. S. LEE   1
J. S. LEE ASSOCIATES, INC.
2001 JEFFERSON DAVIS HIGHWAY
SUITE 802
ARLINGTON
VA   22202

PROF. L. C. LEE   1
DEPARTMENT OF STATISTICS
VIRGINIA POLYTECHNIC INSTITUTE
 AND STATE UNIVERSITY
BLACKSBURG
VA   24061

PROF. R. S. LEVENWORTH   1
DEPT. OF INDUSTRIAL AND SYSTEMS
ENGINEERING
UNIVERSITY OF FLORIDA
GAINSVILLE,
FLORIDA 32611

25

DISTRIBUTION LIST

1

PROF G. LIEBERMAN
STANFORD UNIVERSITY
DEPARTMENT OF OPERATIONS RESEARCH

STANFORD CALIFORNIA 94305

1

DR. JAMES R. MAAR
NATIONAL SECURITY AGENCY
FORT MEADE , MARYLAND
20755

1

PROF. R. W. MADSEN
DEPARTMENT OF STATISTICS
UNIVERSITY OF MISSOURI
COLUMBIA
MO
65201

1

DR. N. R. MANN
SCIENCE CENTER
ROCKWELL INTERNATIONAL CORPORATION
P.O. BOX 1085
THOUSAND OAKS ,
CALIFORNIA 91360

1

DR. W. H. MARLOW
PROGRAM IN LOGISTICS
THE GEORGE WASHINGTON UNIVERSITY
707 22ND STREET , N. W.
WASHINGTON , D. C.
20037

1

PROF. E. MASRY
DEPT. APPLIED PHYSICS AND
INFORMATION SERVICE
UNIVERSITY OF CALIFORNIA
LA JOLLA
CALIFORNIA 92093

1

DR. BRUCE J. MCDONALD
SCIENTIFIC DIRECTOR
SCIENTIFIC LIAISON GROUP
OFFICE OF NAVAL RESEARCH
AMERICAN EMBASSY - TOKYO
AFO SAN FRANCISCO 96503

DR. JANET M. MYHRE   1
THE INSTITUTE OF DECISION SCIENCE
   FOR BUSINESS AND PUBLIC POLICY
CLAREMONT MEN'S COLLEGE
CLAREMONT
CA   91711

MR. H. NISSELSON   1
BUREAU OF THE CENSUS
ROOM 2025
FEDERAL BUILDING 3
WASHINGTON ,
D. C.   20233

MISS B. S. ORLEANS   1
NAVAL SEA SYSTEMS COMMAND
(SEA 03F)
RM 10S08
ARLINGTON VIRGINIA   20360

PROF. D. E OWEN   1
DEPARTMENT OF STATISTICS
SOUTHERN METHODIST UNIVERSITY
DALLAS
TEXAS
75222

PROF. E. PARZEN   1
STATISTICAL SCIENCE DIVISION
STATE UNIVERSITY OF NEW YORK
AT BUFFALO
AMHERST
NEW YORK   14226

DR. A. PETRASOVITS   1
ROOM 207B , FOOD AND DRUG BLDG.
TUNNEY'S PASTURE
OTTOWA , ONTARIO K1A-0L2 ,
CANADA

PROF. S. L. PHOENIX   1
SIBLEY SCHOOL OF MECHANICAL AND
   AEROSPACE ENGINEERING
CORNELL UNIVERSITY
ITHACA
NY   14850

DR. A. L. POWELL   1
DIRECTOR
OFFICE OF NAVAL RESEARCH BRANCH OFF
495 SUMMER STREET
BOSTON
MA   02210

MR. F. R. PRICFI   1
CODE 224 OPERATIONSL TEST AND ONRS
EVALUATION FORCE (OPTEVFOR)
NORFOLK ,
VIRGINIA
20360

PROF. M. L. PURI
DEPT. OF MATHEMATICS
P.O. BOX F
INDIANA UNIVERSITY FOUNDATION
BLOOMINGTON
IN　　　　　　　　　47401　　　　　　　　1

PROF. H ROBBINS
DEPARTMENT OF MATHEMATICS
COLUMBIA UNIVERSITY
NEW YORK,
NEW YORK 10027　　　　　　　　　　　　1

PROF. M ROSENBLATT
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF CALIFORNIA SAN DIEGO
LA JOLLA
CALIFORNIA　　　　　92093　　　　　　　1

PROF. S. M. ROSS
COLLEGE OF ENGINEERING
UNIVERSITY OF CALIFORNIA
BERKELEY
CA　　　　　　　　94720　　　　　　　　1

PROF. I RUBIN
SCHOOL OF ENGINEERING AND APPLIED
SCIENCE
UNIVERSITY OF CALIFORNIA
LOS ANGELES ,
CALIFORNIA 90024　　　　　　　　　　　1

PROF. I. R. SAVAGE
DEPARTMENT OF STATISTICS
YALE UNIVERSITY
NEW HAVEN,
CONNECTICUT
06520　　　　　　　　　　　　　　　　1

PROF. L. L. SCHARF JR
DEPARTMENT OF ELECRICAL ENGINEERING
COLORADO STATE UNIVERSITY
FT. COLLINS ,
COLORADO
80521　　　　　　　　　　　　　　　　1

PROF. R. SERFLING
DEPARTMENT OF STATISTICS
FLORIDA STATE UNIVERSITY

TALLAHASSEE　FLORIDA　32306　　　　　　1

PROF. W. R. SCHUCANY
DEPARTMENT OF STATISTICS
SOUTHERN METHODIST UNIVERSITY
DALLAS ,
TEXAS
75222　　　　　　　　　　　　　　　　1

PROF. C. C. SIEGMUND   1
DEPT. OF STATISTICS
STANFORD UNIVERSITY
STANFORD
CA   94305

PROF. M. L. SHOOMAN   1
DEPT. OF ELECTRICAL ENGINEERING
POLYTECHNIC INSTITUTE OF NEW YORK
BROOKLYN,
NEW YORK
11201

DR. A. L. SLAFKOSKY   1
SCIENTIFIC ADVISOR
COMMANDANT OF THE MARINE CORPS
WASHINGTON,
D. C.
20380

DR. C. E. SMITH   1
DESMATICS   INC.
P. O. BOX 618
STATE COLLEGE
PENNSYLVANIA
16801

PROF. W. L. SMITH   1
DEPARTMENT OF STATISTICS
UNIVERSITY OF NORTH CAROLINA
CHAPEL HILL
NORTH CAROLINA 27514

Dr. H. J. Solomon   1
ONR
223/231 Old Marylebone Rd
London NW1 5TH, ENGLAND

MR. GLENN F. STAHLY   1
NATIONAL SECURITY AGENCY
FORT MEADE,
MARYLAND 20755

Mr. J. Gallagher   1
Naval Underwater Systems Center
New London, CT

Dr. E. C. Monahan   1
Dept. of Oceanography
University College
Galway, Ireland

29

No. of Copies

DR. R. M. STARK
STATISTICS AND COMPUTER SCI.
UNIV. OF DELAWARE
NEWARK
DELAWARE
19711   1

PROF. RICHARD VANSLYKE
RES. ANALYSIS CORP.
BEECHWOOD
OLD TAPPEN ROAD
GLEN COVE, NEW YORK
11542   1

PROF. JOHN W. TUKEY
FINE HALL
PRINCETON UNIV.
PRINCETON
NEW JERSEY
08540   1

DR. THOMAS C. VARLEY
OFFICE OF NAVAL RESEARCH
CODE 434
ARLINGTON
VA
22217   1

PROF. G. WATSON
FINE HALL
PRINCETON UNIV.
PRINCETON
NEW JERSEY
C8540   1

MR. DAVID A. SWICK
ADVANCED PROJECTS GROUP
CODE 8103
NAVAL RESEARCH LAB.
WASHINGTON
DC   20375   1

MR. WENDELL G. SYKES
ARTHUR D. LITTLE, INC.
ACORN PARK
CAMBRIDGE
MA
02140   1

PROF. J. R. THOMPSON
DEPARTMENT OF MATHEMATICAL SCIENCE
RICE UNIVERSITY
HOUSTON.
TEXAS
77001   1

PROF. W. A. THOMPSON
DEPARTMENT OF STATISTICS
UNIVERSITY OF MISSOURI
COLUMBIA,
MISSOURI
65201   1

No. of Copies

PROF. F. A. TILLMAN
DEPT. OF INDUSTRIAL ENGINEERING
KANSAS STATE UNIVERSITY
MANHATTAN
KS
    66506 1

PROF J. W. TUKEY
DEPARTMENT OF STATISTICS
PRINCETON UNIVERSITY

PRINCETON , N. J.   08540 1

PROF. A . F . VEINOTT
DEPARTMENT OF OPERATIONS RESEARCH
STANFORD UNIVERSITY
STANFORD
CALIFORNIA
94305 1

DANIEL H. WAGNER
STATION SQUARE ONE
PAOLI , PENNSYLVANIA
19301 1

PROF. GRACE WAHBA
DEPT. OF STATISTICS
UNIVERSITY OF WISCONSIN
MADISON
WI
    53706 1

PROF. K. T. WALLENIUS
DEPARTMENT OF MATHEMATICAL SCIENCES.
CLEMSON UNIVERSITY
CLEMSON,
SOUTH CAROLINA 29631 1

PROF. G. S. WATSON
DEPARTMENT OF STATISTICS
PRINCETON , N. J.   08540 1

PROF. BERNARD WIDROW
STANFORD ELECTRONICS LAB
STANFORD UNIVERSITY
STANFORD
CA
    94305 1

31

DISTRIBUTION LIST

No. of Copies

PROF. G. E. WHITEHOUSE
DEPT. OF INDUSTRIAL ENGINEERING
LEHIGH UNIVERSITY
BETHLEHEM
PA

18015

1

PROF. S. ZACKS
DEPT. OF MATHEMATICS AND STATISTICS
CASE WESTERN RESERVE UNIVERSITY
CLEVELAND.,
OHIO
44106

1

PROF. M. ZIA-HASSAN
DEPARTMENT OF INDUSTRIAL AND
   SYSTEMS ENGINEERING
ILLINOIS INSTITUTE OF TECHNOLOGY
CHICAGO
IL

60616

1

HEAD, MATH. SCI SECTION
NAT. SCIENCE FOUNDATION
1800 G STREET, N.W.
WASHINGTON, D.C.

20550

1

Dr. L. Rouse
Coastal Studies Institute
Louisiana State University
Baton Rouge, LA

1

Dr. H. Sittrop
Physics Lab., TNO
P.O. Box 96964
2509 JG, The Hague
The Netherlands

DR. R. ELASHOFF
BIOMATHEMATICS
UNIV. OF CALIF.
LOS ANGELES
CALIFORNIA

90024

1

PROF. GEORGE S. FISHMAN
UNIV. OF NORTH CAROLINA
CUR. IN OR AND SYS. ANALYSIS
PHILLIPS ANNEX
CHAPEL HILL, NORTH CAROLINA

20742

1

DR. R. GNANADESIKAN
BELL TELEPHONE LAB
HOLMDEL, N. J.

07733

1

32

1

DR. A. J. GOLEMAN
CHIEF, CR
DIV. 2C5.C2, ADMIN. A428
U.S. DEPT. OF COMMERCE
WASHINGTON, D.C.

20234

1

DR. H. HIGGINS
53 BONN 1, POSTFACH 585
NASSESTRASSE 2

WEST GERMANY

1

DR. P. T. HOLMES
DEPT. OF MATH.
CLEMSON UNIV.
CLEMSON
SOUTH CAROLINA

29631

1

CR. J. A. HOCKI
BELL TELEPHONE LABS
HOLMDEL
NEW JERSEY

07733

1

DR. ROBERT HOOKE
MATH. DEPT.
WESTINGHOUSE RES. LABS
CHURCHILL BORO
PITTSBURGH, PENNSYLVANIA

15235

1

DR. D. L. IGLEHART
DEPT. OF C.R.
STANFORD UNIV.
STANFORD
CALIFORNIA

94305

1

Dr. D. Trizna, Mail Code 5323
Naval Research Lab
Washington, D.C. 20375

1

Dr. E. J. Wegman,
ONR, Cdoe 436
Arlington, VA 22217

33

DISTRIBUTION LIST

No. of Copies

DR. H. KOBAYASHI   1
IBM
YORKTOWN HEIGHTS
NEW YORK
                                        10598

DR. JOHN LEHOCZKY   1
STATISTICS DEPARTMENT
CARNEGIE-MELLON UNIVERSITY
PITTSBURGH
PENNSYLVANIA
                                        15213

DR. A. LEMOINE   1
1020 GUINDA ST.
PALO ALTO,
CALIFORNIA
                                        94301

DR. J. MACCUEEN   1
UNIV. OF CALIF.
LOS ANGELES
CALIFORNIA
                                        90024

PROF. K. T. MARSHALL   1
DEPT. OF OR
NAVAL POSTGRADUATE SCHOOL
MONTEREY
CALIFORNIA
                                        93940

DR. M. MAZUMDAR   1
MATH. DEPT.
WESTINGHOUSE RES. LABS
CHURCHILL BORO
PITTSBURGH
PENNSYLVANIA   15235

DR. LEON F. MCGINNIS   1
SCHOOL OF IND. AND SYS. ENG.
GEORGIA INST. OF TECH.
ATLANTA
GEORGIA
                                        30332

Dr. K. A. Ulbricht,   1
DFVLT
Inst. for Comm. Tech.
German Aerospace Res. Estab.
8931 Webling,
Oberfhaffenhoven, West Germany

34

DR. D. R. MCNEIL   1
DEPT. OF STATISTICS
PRINCETON UNIV.
PRINCETON
NEW JERSEY   08540

DR. F. MOSTELLER   1
STAT. DEPT.
HARVARD UNIV.
CAMBRIDGE
MASSACHUSETTS   02139

DR. M. REISER   1
IBM
THOMAS J. WATSON RES. CTR.
YORKTOWN HEIGHTS
NEW YORK   10598

DR. J. RIORDAN   1
DEPT. OF MATHEMATICS
ROCKEFELLER UNIV.
NEW YORK
NEW YORK   10021

DR. LINUS SCHRAGE   1
UNIV. OF CHICAGO
GRAD. SCHOOL OF BUS.
5836 GREENWOOD AVE.
CHICAGO, ILLINOIS   60637

DR. PAUL SCHWEITZER   1
THOMAS J. WATSON RESEARCH CTR9
POST OFFICE BOX 218
YORKTOWN HEIGHTS
NEW YORK   10598

DR. RICHARD SORENSON   1
CODE 303 NPRDC
271 CATALINA BLVD.
SAN DIEGO
CALIFORNIA   92152

DR. V. SRINIVASAN   1
GRADUATE SCHOOL OF BUSINESS
STANFORD UNIVERSITY
STANFORD
CALIFORNIA   94305

Dr. Roy Welsch   1
M.I.T. SLOAN SCHOOL
Cambridge, MA 02139